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Magnetically Stabilized Fluidized Beds for Solids Separation by Density

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Abstract

Systems for the dry separation of solids by density difference are described. They consist of a magnetically stabilized fluidized bed (MSB) as a host medium in which heavy solids sink and light solids float. The magnetic stabilization produces a fluidized medium with the absence of gas bubbling and thus enhances the separations efficiency by preventing remixing of the feed solids to be separated. The MSB can be maintained with an apparent bulk density between that of solids in a mixture to be separated by adjusting the gas fluidization velocity. Several experiments have been conducted using both batch and continuous MSBs to separate a solids mixture consisting of coal and limestone. In the continuously operated experiments using this binary feed, recoveries of 90% of the feed solids at over 90% purity were obtained. Preferred operating conditions for the MSB are discussed which result in minimizing the effects of bed yield stress and viscosity on the separations efficiency. Configurations for the separation of multicomponent feeds are also proposed.

INTRODUCTION

There are many devices available for the separation of solids mixtures according to their density (*1*). These include, for example, vibrating screens, fluidized bed classifiers, and liquid flotation devices. Currently, the most practical and efficient industrial separation process for solids by

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density difference is the wet process using a liquid as the sorting medium where solids heavier than the liquid sink and those lighter than the liquid float (*2*). A magnetic fluid concept employing controllable apparent specific gravity is also being developed (*3*).

For a variety of reasons, there exists a need for a dry solids separation process. This need arises from such problems in wet processes as: the lack of liquid availability (e.g., water shortage in arid areas); the unsuitability of wet processes (e.g., some oil shales are difficult to process with liquids); environmental concerns (e.g., water pollution problems with almost permanent suspension of very fine coal particles); and the energy requirement for redrying after wet operations.

Among the dry processes for solids separation by density, operations employing pneumatic jigs and cyclones are relatively common for small-scale separations or where the solids are present in small concentrations. These dry processes, however, are either inefficient in terms of separation sharpness or insufficient in terms of operating capacity when compared with most wet process operations.

In another category, fluidized beds can also be utilized for solids separation (*4*). The use of fluidized beds for solids separation appears to be the first proposed use for the fluidization phenomenon, predating its use in the chemical and petroleum areas (*5*). These classifiers are of two general types: those where the fluidized bed itself is made up of the solids to be separated and those where the fluidized bed acts as a host medium for other solids to be separated. This technology is not presently used in commercial processes, however, because of such problems as intermixing of the solids to be separated due to the bubbling turbulence.

Recently, there has been renewed interest in the use of fluidized beds for solids separation (*6–10*). Normal fluidization continues to be limited in operating flexibility due to the presence of gas bubbles. The mixing motions cause a decrease in the sharpness of the separation as well as a reduction in capacity. This has resulted in operations being restricted to gas velocities near the minimum fluidization velocity, so as to mitigate the effects of large gas bubbles which would be present at higher fluidization velocities. The bubbling limitation can be overcome and other benefits achieved through the use of a magnetically stabilized fluidized bed (MSB). The MSB provides a fluidized bed host medium with the absence of gas bubbles. As a result, the solids backmixing which limited the previous devices is eliminated. In addition, the fluidization velocity can be maintained over a broad range in the stabilized bed, allowing a controlled variability in the host medium density.

MAGNETICALLY STABILIZED BEDS

The quiescent, fluidlike MSB is free of bubbles or pulsations which are normally present in fluidized beds and can be established when a magnetic field is applied to a bed of fluidizable, magnetizable particles (*11–13*). A graphical comparison of a conventional fluidized bed and an MSB is shown in Fig. 1. The magnetic stabilization produces a nonbubbling fluid state with operating velocities ranging between the normal minimum fluidization velocity in the absence of the applied magnetic field and an upper limit given by the superficial fluid velocity required to cause bubbling to occur in the bed. The stably fluidized particles behave in many ways as a liquid and are easily transported into and out of process vessels. For example, light nonmagnetic objects float and heavy ones sink when placed in the bed, and the medium discharges through an orifice from the process vessel with a well-defined discharge coefficient.

Figure 2 shows schematically that the fluid/solid/magnetic system exists in one of three regimes. Below the minimum fluidization velocity

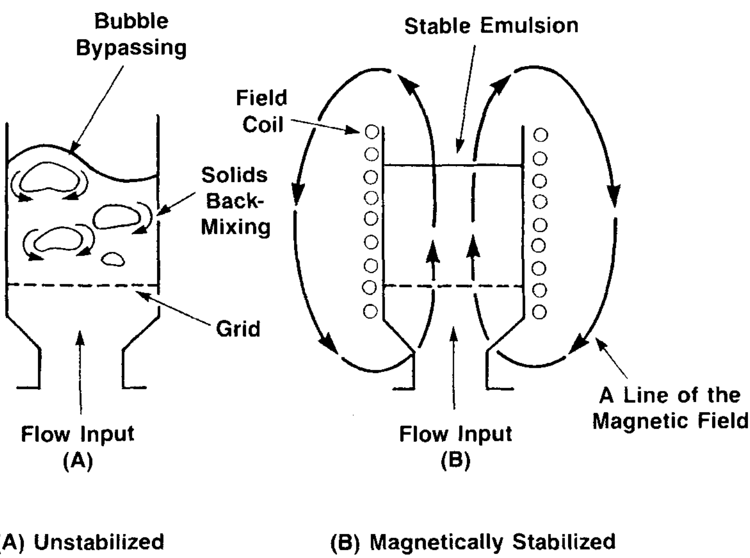

FIG. 1. Comparison of bubbling and stabilized fluidized states.

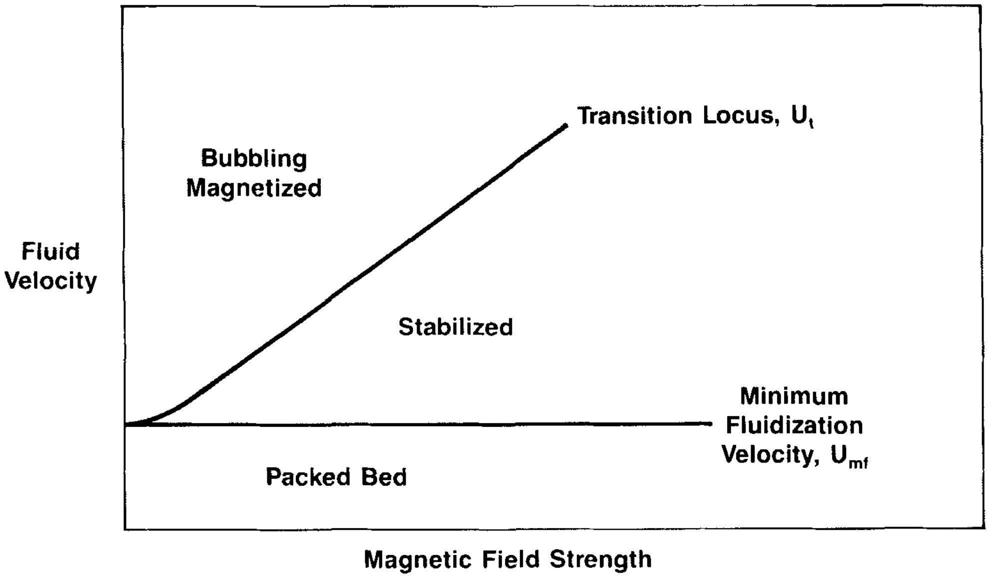

FIG. 2. Phase diagram of fluid/solids/magnetic phenomena.

the pressure drop across the bed is less than the bed weight per unit area and the bed has the structure and density of a packed bed. In the stabilized regime the bed is expanded but is free of mixing motions. In the regime denoted "bubbling magnetized," the bed bubbles even though magnetized. The transition between these regimes is sharp and reproducible and is a function of the specific fluid-particle system, among other things. The MSB regime existing at velocities between the minimum fluidization velocity and the transition velocity to bubbling is the one of interest in performing separations of solids by density difference in the work described herein.

When increasing the gas velocity in an MSB, as shown schematically in Fig. 3, the breakpoint of the pressure drop curve corresponds to the minimum fluidization velocity, U_{mf} (for pressure drop data of an actual bed, see Fig. 9.10 of Ref. *3*). Thereafter, as in unmagnetized bubbling fluidized beds, the pressure drop through the MSB equals the weight of bed particles per unit bed area independent of particle size or gas fluidization velocity. This remains true unless gas velocity is reduced, in which case a hysteresis oftentimes exists and the pressure drop is lower at a given velocity than when increasing the flow. This phenomenon is not

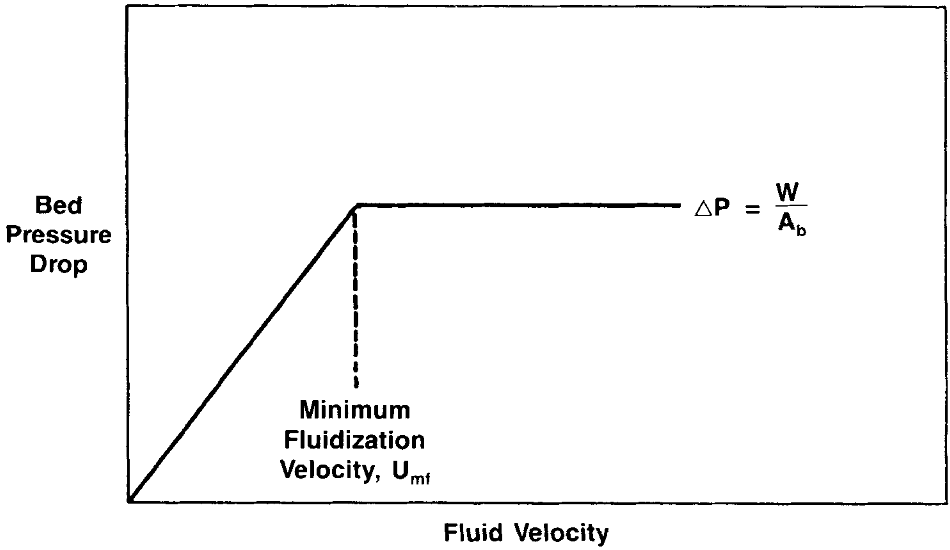

FIG. 3. Pressure drop behavior of magnetically stabilized fluidized beds.

critical to the sink–float application. However, bed length and hence apparent bed density exhibit a similar hysteresis which should be taken into consideration when operating an MSB for solids separation.

SOLIDS SEPARATION RESULTS

We have conducted numerous experiments to demonstrate the feasibility of separating solids by density difference in magnetically stabilized beds. These experiments have ranged from simple sink–float tests in batch units to tests in flowing MSBs with continuous solids feed.

Batch Units

Initially a simple test using a batch MSB consisting of −60+80 U.S. Sieve magnetite was used to demonstrate the potential solids separating ability of the MSB. Using a magnetic field intensity of 32 oersted and a gas velocity of 45 cm/s, the bulk specific gravity (S.G.) of the MSB was maintained at 1.47. The feed mixture of solids to be separated contained coal (S.G. 1.39) and limestone (S.G. 2.71), each approximately 0.65 cm in size. These pieces of coal and limestone were introduced into the MSB by dropping them from a standard height of 20 cm above the bed top surface. It was observed that coal always floated on the top surface of the MSB, while the limestone always disappeared into the bed. This result was expected since the MSB maintained an effective bulk density intermediate to that of the coal and the limestone. The transition velocity of the MSB under these operating conditions was 4.9 cm/s. Other tests achieved the same separation with an MSB operated at a S.G. of 1.73.

Experiments using a batch MSB consisting of −140+200 U.S. Sieve iron spheres were used to obtain quantitative data on the separation of solids with a broader density range. The bed was maintained at a bulk density of about 4.2 g/cm^3 using a fluidizing gas velocity of 5.2 cm/s with an applied ac magnetic field having an rms intensity of 16.2 oersted. Solids with a diameter of 0.95 cm, which ranged in density from below 3 to above 10 g/cm^3, were added to the bed. These solids were prepared from hollow aluminum spheres filled with different amounts of lead. After 60 min of operating time, the flow of fluidizing gas was stopped and the magnetic field removed. The location of each of the feed solids in the bed was determined, giving the results shown in Fig. 4. While only some of the solids with a density below that of the bulk density of the MSB were found to float, all of those solids with a higher density sank below the surface. The depth of the sink solids was found to increase monotonically with the difference in their density to that of the MSB. Further study is required to determine the extent to which this phenomenon is a rate or equilibrium process.

Figure 4 indicates, as mentioned, that some of the solids with a specific gravity less than that of the MSB did in fact sink into the bed. It has previously been reported, however, that a vertical variation in density exists within a stabilized bed, with lower density in a zone adjacent to the top surface of the bed (*14*). This would explain the sinking of these solids which are less than, but close to, the average bed bulk density.

Other experiments were conducted using a batch MSB to determine more fully the range of operating limits at which coal test solids placed on the bed top surface would sink into the bed. The MSB particles used were −10+20 U.S. Sieve composites of stainless steel in alumina with an overall density of 1.8 g/cm^3, and the coal solids ranged in size from 0.40 to 0.95 cm. The MSB was operated at several different combinations of fluidizing gas velocity and magnetic field strength. These results are given in Fig. 5 where the conditions under which the coal sinks are shown as open squares and the conditions under which the coal did not sink are shown as filled triangles. It is seen that the limits to MSB sink–float

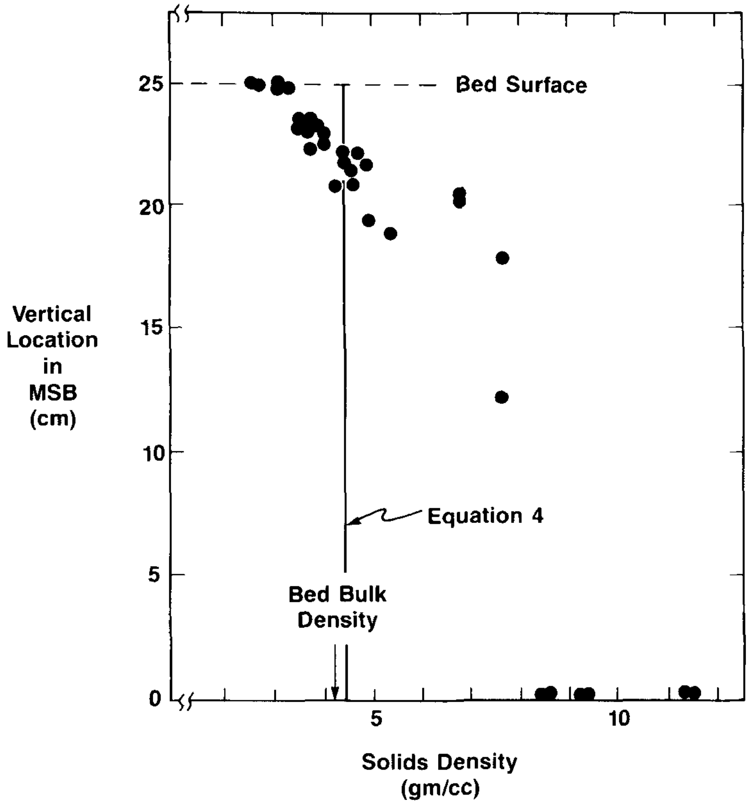

FIG. 4. Stratification of solids by density in a magnetically stabilized fluidized bed.

operation depend on both the magnetic field strength and the fluidizing gas velocity, with preferred conditions being at higher gas velocities as the magnetic field is increased.

Hysteresis Measurements

Other tests were performed to determine the influence of the fluidizing gas velocity on the resolution of the solids separation. A hydrometer-type float was fabricated from a hollow tube closed at the bottom end, weighted with nonmagnetic beads, and affixed with a scale calibrated for specific gravity using a number of liquids of known density. When the

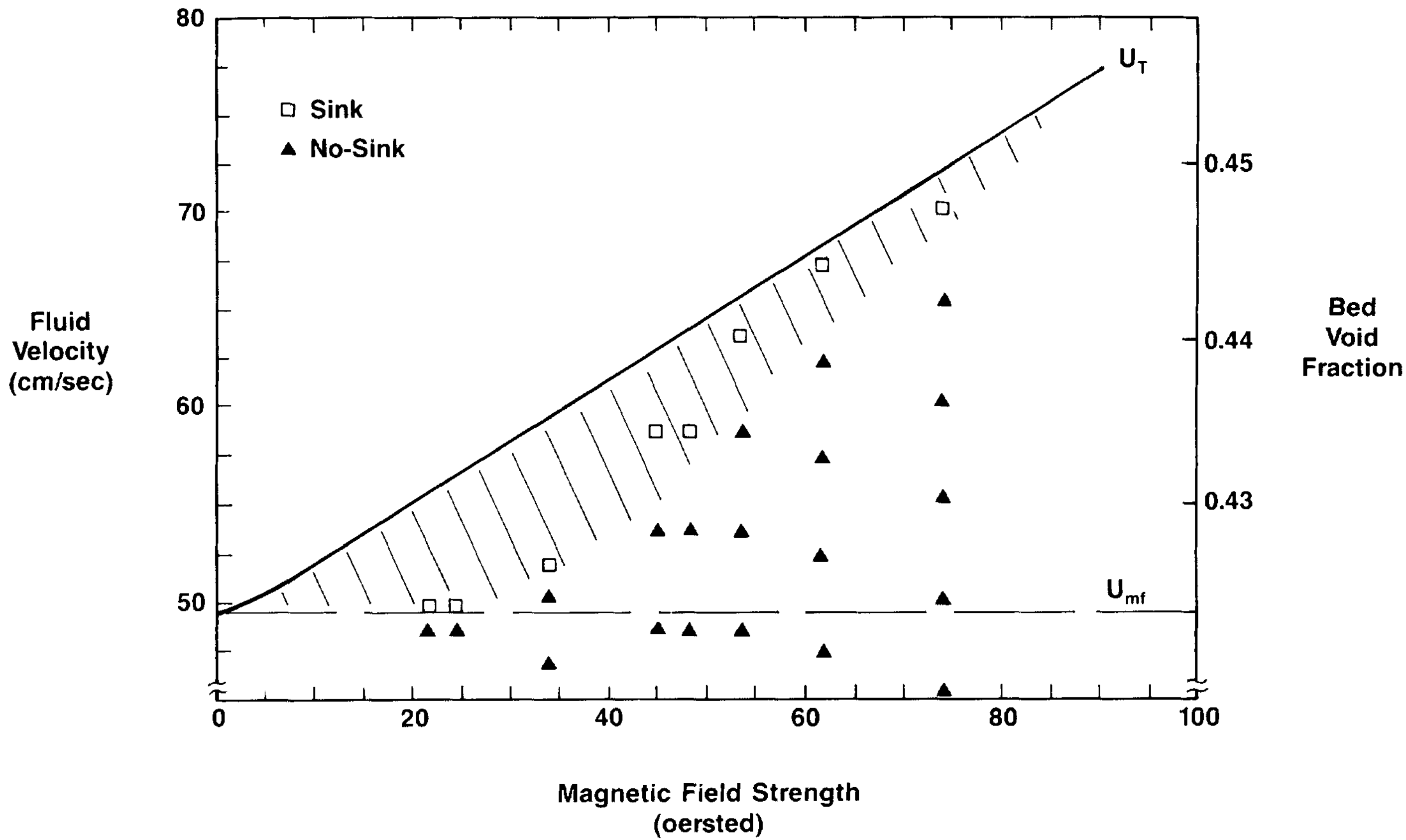

FIG. 5. Operability limits in MSB region from coal solids experiments.

hydrometer was placed in an MSB, the scale reading indicated the apparent specific gravity of the bed medium. Due to the previously reported existence of yield stresses in an MSB (*15*), a hysteresis is expected to exist, especially at conditions removed from the transition between the stabilized and bubbling regime.

Two procedures were used for these experiments. In one, the hydrometer was vertically submerged until its bottom end was in contact with the fluid distributor grid at the bottom of the bed. It was then released and allowed to rise to its equilibrium position. The scale readings of apparent specific gravity obtained in this manner were denoted the "float" reading. Alternatively, the hydrometer was released with its bottom end at the top surface of the bed and allowed to settle to an alternate equilibrium position. This reading was denoted the "sink" specific gravity.

Figure 6 presents hysteresis measurements for an MSB of −70+100 U.S. Sieve steel particles in an applied magnetic field of 30 oersteds. "Sink" specific gravity always equaled or exceeded "float" specific gravity as long as the bed was operated at gas velocities below that at transition, U_t. The difference between the two readings is greatest in the stably fluidized regime at an operating velocity just at or above U_{mf}, the minimum fluidization velocity. The difference diminishes as superficial velocity increases and asymptotically approaches zero near the bubbling transition velocity, U_t. These hydrometer tests indicate that the operation of MSBs for solids separation should preferably be practiced at gas velocities near to but less than the transition velocity.

The data in Fig. 6 indicate that solid objects lighter than the bed tend to rise upward through the bed regardless of the superficial velocity (i.e., distance from the transition to bubbling). Solids heavier than the bed, however, might not always sink since their apparent specific gravity is a function of operating conditions. Thus, the "sinks" or heavier solids tend to become lodged in the bed while the "floats" or lighter solids do not. One preferred operating configuration, therefore, would classify the feed solids by having them float selectively to the surface of an MSB rather than sinking to the bottom.

Continuous Units

In order to apply MSBs to the continuous separation of solids at high feed rates, it is advantageous to operate the MSB as a moving bed. Two possible configurations for these systems are vertical downflow and crossflow, as shown in Figs. 7 and 8. In the downflow configuration, feed solids may be introduced on or below the bed surface. Introduced below

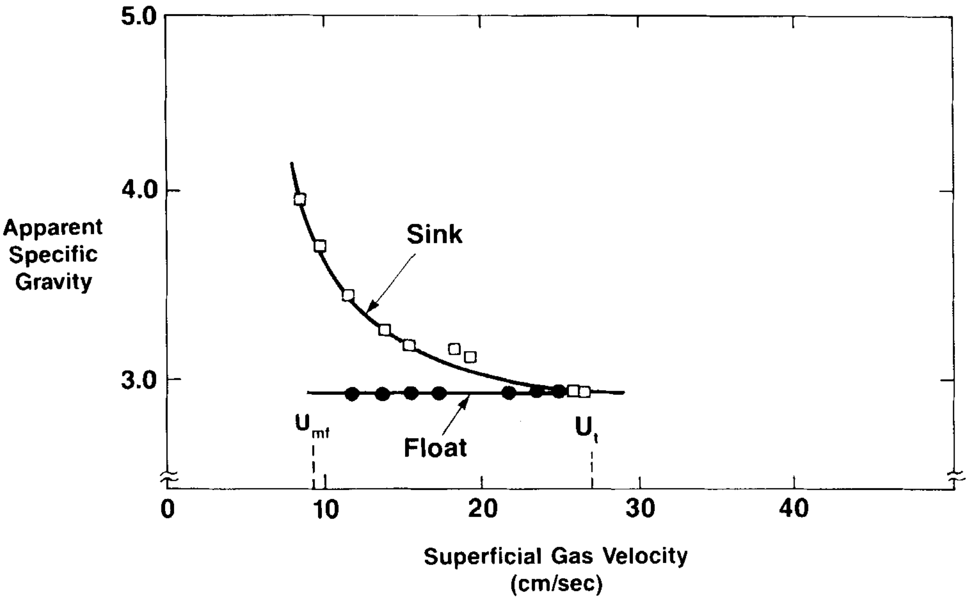

FIG. 6. Results of hysteresis measurements.

the surface, the light solids rise in the bed and are collected at the surface. Heavy solids are conveyed downward with the bed and thereby removed and recovered. In the crossflow configuration shown in Fig. 8, the MSB moves transversely relative to an upwardly moving fluidizing gas. Solids to be separated sink or float while being carried horizontally with the bed to a recovery zone. In the crossflow configuration the solids may be introduced on the top of the bed as shown in Fig. 8 or within the bed itself. Introducing solids near the bottom of either bed takes advantage of the relative ease with which solids rise in the MSB compared to their sinking as was found by the hysteresis experiments previously discussed.

A crossflow MSB 70 cm long, 5.1 cm high, and 2.54 cm wide containing −14+20 U.S. Sieve composite particles of 70 wt% stainless steel and 30% alumina was used to perform a continuous solids separation. The bed was fluidized with air at a superficial gas velocity of about 109 cm/s in an applied dc magnetic field of 75 oersted. The field source was a pair of air core current-carrying coils wound on separate elliptical spools having the major inside axis larger than the bed length. The coils were placed around the bed and oriented horizontally with a common axis and

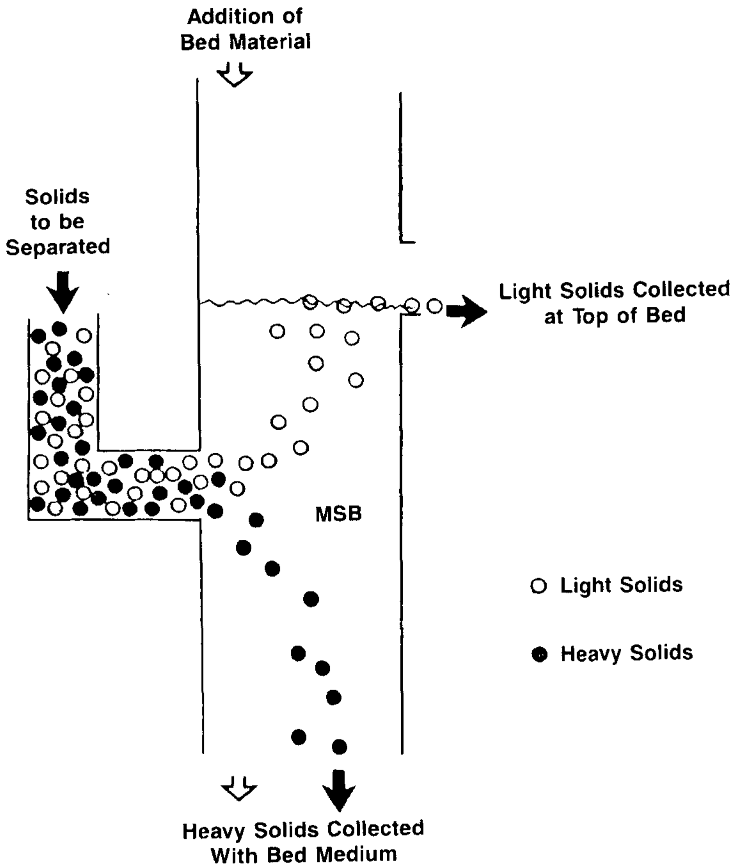

FIG. 7. Continuous vertically moving magnetically stabilized fluidized bed for solids/solids separation.

orientation to provide a nearly uniform vertical field. In operation, the expanded, bubble-free bed moved horizontally along the length of the vessel with some inclination of the bed top surface. The top surface became more level when superficial velocity was increased, a trend previously reported (*16*).

Feed solids were a mixture of coal (1.39 g/cm^3) as light component and limestone (2.71 g/cm^3) as the heavy component. The size of the feed solids ranged from 0.4 to 1.3 cm.

In operation the bed particles were added continuously at the top of

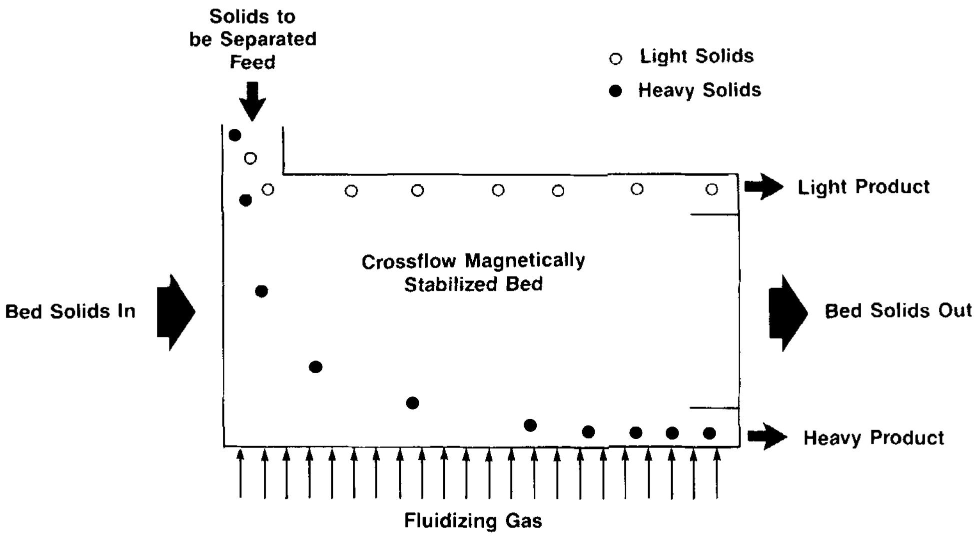

FIG. 8. Continuous crossflow magnetically stabilized fluidized bed for solids/solids separation.

one end of the bed, removed from the other end, and recirculated to the entrance with an external pneumatic conveying loop. The exit end of the bed was configured with two partitions, one at the top of the end and another at the bottom of the end, to remove bed material containing the light and heavy solids, respectively. Both product streams were separated from the bed particles continuously using screens external to the bed. As shown in Table 1, a feed mixture of nearly equal weight of coal and limestone was separated into a coal-rich float product and a lime-rich sink product. Both product streams were considerably enriched in their major component and the recoveries were good. It is expected that the performance of the process can be improved or optimized using different operating conditions or bed particles.

DISCUSSION

The preceding descriptions, including the separation of coal and limestone in both batch and continuous flowing magnetically stabilized fluidized beds, clearly demonstrate the potential utility of the concept for application. By proper selection of the bed particles and fluid velocity the bulk density of the MSB can be adjusted to provide an appropriate value intermediate to those of the desired sink and float solids to be separated.

TABLE 1
Results of Solids Separation Experiments in a Crossflow MSB

Feed		Light product		Heavy product	
Coal	148.8 g (50 wt%)	Coal	134.8 g (97 wt%)	Coal	10.6 g (7 wt%)
Lime	149.5 g (50 wt%)	Lime	4 g (3 wt%)	Lime	145.7 g (93 wt%)
	298.3 g		138.8 g		156.3 g
		Remaining in bed:	0.3 g		
			295.4 g		

Material balance: 99%

% Recovery of light in light product (coal) based on feed amount: 90.6%
% Recovery of heavy in heavy product (lime) based on feed amount: 97.5%

For example, Fig. 9 shows an MSB phase diagram for −60+80 U.S. Sieve steel spheres with the variation in bulk bed density indicated in the stabilized regime. Increasing the fluid velocity decreases the bulk density of the bed because the bed expands. Because of bed rheological properties, simple sink/float behavior is not always attained. Nonetheless, proper selection of the applied field intensity will insure fluidity of the bed (*17*) to permit stratification of solids according to their density, which in turn permits subsequent separation.

Separating Multicomponent Mixtures

While separation of binary mixtures has wide application, there are other cases where it may be beneficial to produce a multiplicity of product solids of different densities. Figures 10 and 11 show two potential configurations for such operation. In Fig. 10, a single continuous crossflow MSB is used with bed flow transverse to the ascending flow of the fluidizing fluid. Solids to be separated are introduced at the bottom of the bed. These feed solids rise to the top surface of the bed at a rate which is a function of the difference of their density to the apparent bulk density of the MSB. Solids which are much lighter than the MSB and thus have a high density difference rise more quickly through the bed. They are, therefore, transported horizontally only short distances from the solids feed point. Solids which are heavier and may be close in density to that of the MSB rise more slowly through the bed and are, therefore, transported much further horizontally from the solids feed point. Solids of inter-

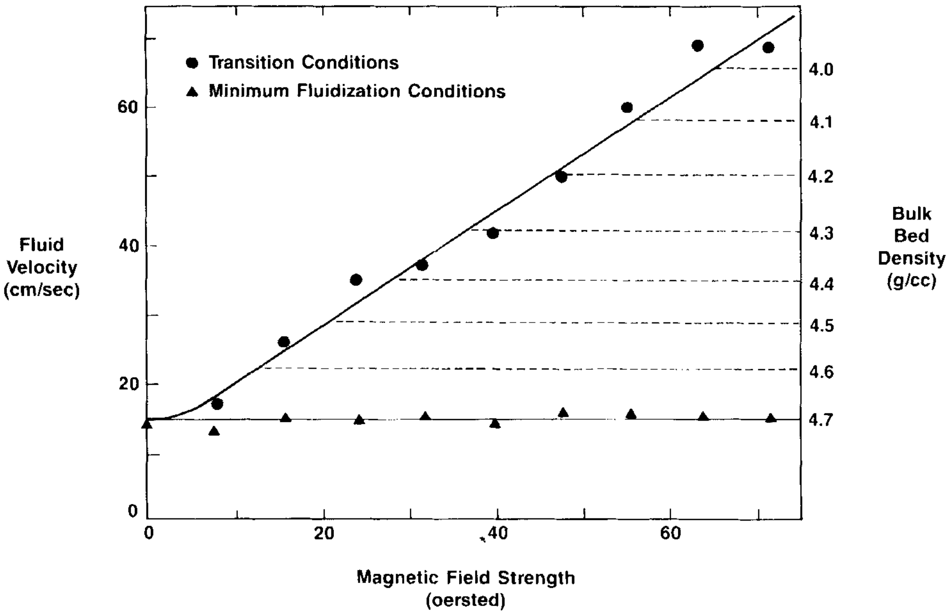

FIG. 9. Variation in bed bulk density with fluidization velocity.

mediate densities are recovered at intermediate locations along the top surface of the MSB and may be removed by a solids conveyer system. Because the separation mechanism in this configuration depends on the rate of solids ascent, the separating ability of the MSB depends on the factors affecting drag (e.g., MSB viscosity, solids size, solids shape, etc.) and not just on the density difference.

Another possible configuration for the separation of a multiplicity of solids is shown in Fig. 11. In this configuration a number of separate MSBs are maintained at different bed densities by using different gas velocities for each bed. The bed solids sequentially flow into stages of increasing apparent density. The increase in MSB apparent density is obtained by decreasing the gas velocity to the bed. Thus, the first or uppermost bed has the highest fluid velocity and thus the lowest bulk density. Only the lightest solids in the feed stay on the top surface of the first bed to be recovered. In subsequent beds, heavier and heavier components of the feed remain on the top of the bed to be recovered. An advantage of this configuration is that the individual MSBs can be kept relatively shallow. It has previously been reported that the yield stress

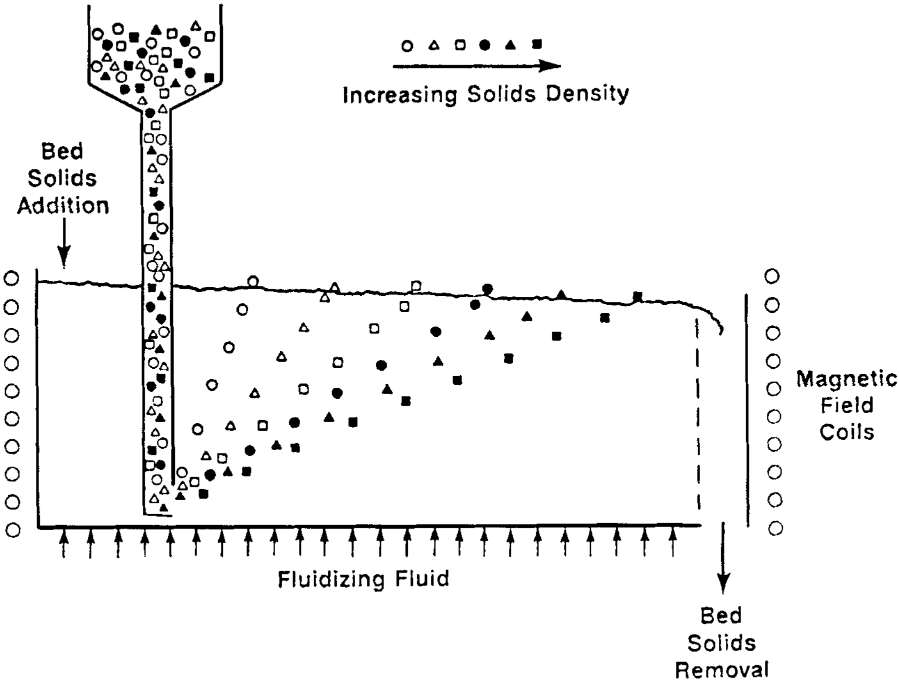

FIG. 10. Crossflow MSB device for the separation of a multiplicity of solids by density difference in a single vessel.

within an MSB increases with bed depth (*14, 15*). Keeping the beds shallow would tend to minimize the yield stresses and, therefore, allow sharper separations as discussed below.

Magnetic Field Influence

The strength of the magnetic field to be applied to the fluidized solids in the contacting zone will depend on the magnetizability of the bed particles and the degree of bed stabilization desired. Particles having relatively weak magnetic properties, such as composites and alloys, will require the application of a stronger magnetic field than particles having strong magnetic properties, such as iron, to achieve similar degrees of stabilization.

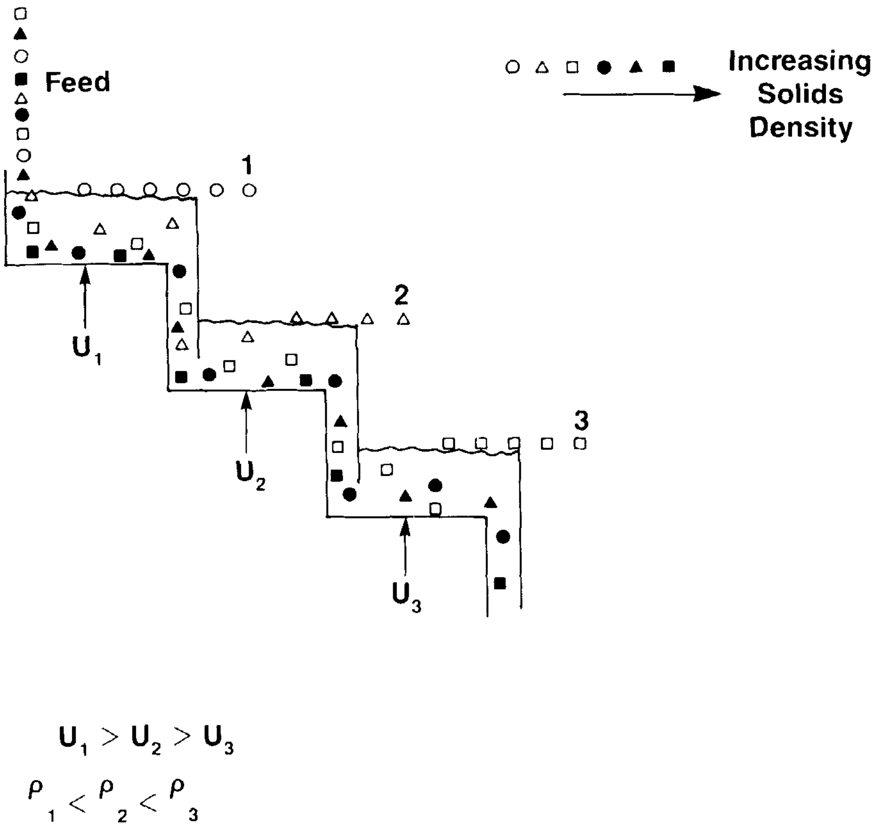

FIG. 11. Multiple crossflow MSB configuration for separation of solids by density difference.

With proper selection of magnetic particles, applied fields are of modest intensity and the power requirement for the magnetic field source for solids separation will be modest. Magnet power dissipation generates heat that generally may be removed using natural convection air cooling. This can eliminate any need for liquid convection cooling and attendant requirements for coolant treatment and recirculation.

The magnetization of the particles should not be so great as to cause excessive particle to particle attractive forces which tend to freeze or lock the bed particles together and prevent separation of the solids due to high yield stress. However, since the strength of the field produced by an electromagnet depends on the amount of current flowing through the coils of the electromagnet, an operator can readily adjust the field strength to achieve the desired degree of stabilization for the particular system employed.

Yield Stress Related to Resolution

As previously mentioned, at some combinations of fluid velocity and magnetic field the yield stress in the bed may become prohibitively high for the operation of an MSB solids separation process. An approximate mathematical relationship can be developed to indicate a quantitative relationship applying to the behavior of the feed solids in an MSB. Thus, an immersed object experiences a buoyant force $gV\Delta\rho$ where g is the gravitational constant, V is the immersed volume, and $\Delta\rho$ is the difference in density between the object and the bed. In addition, fluid viscous drag will produce an upward force on the solid.

The MSB possesses characteristics of a Bingham plastic; a yield stress exists that must be overcome before the bed medium will flow. This yield stress has been reported to be a function of the gas velocity, magnetic field strength, bed depth (*14, 15, 18*), and in continuous units, bed shear rate (*16*). Thus, when the object comes to rest within the bed, a static balance of forces exists with the sum of the buoyant force and fluid drag equal to the yield force required to deform the medium and permit the object to move relative to it. Letting τ_y denote the vertical component of yield stress averaged over the surface of the object, the total yield stress equals $\tau_y A_s$, where A_s is the surface area of the solid. Dynamic forces due to fluidizing gas motion will be neglected. The balance of forces then reads

$$\tau_y A_s \cong gV\Delta\rho \tag{1}$$

For an object of equivalent spherical diameter D, Eq. (1) gives

$$\Delta\rho \cong \frac{6\tau_y}{gD} \tag{2}$$

Equation (2) gives the density difference predicted to produce incipient floating or sinking of an immersed object when yield stress is present. Small density difference $\Delta\rho$ corresponds to high resolution in the sink-float separation process and is desirable. Equation (2), therefore, shows that high resolution is favored by a low value of yield stress τ_y and large object size D. In the separation of materials of different densities, it follows that solids of larger size are more easily separated from each other than solids of smaller size. The decrease of $\Delta\rho$ when the operating velocity approaches the transition velocity U_t (as indicated by the decreasing vertical difference between sink and float curves in Fig. 6) demonstrates that the yield stress is reduced by operating closer to U_t.

Experiments which measure the force required to dislodge a roughened plate positioned vertically in the bed permit a quantitative measurement of yield stress in the MSB. Dimensional reasoning and extensive measurements illustrate that particle size has no influence on the yield stress (*15*). In this manner, an empirical equation has been derived which relates the yield stress within the bed, τ_y, to the particle magnetization, M_p, and the bed void fraction, ε_0 (*3*):

$$\tau_y = \frac{AM_p^2}{4\pi\varepsilon_0^{12}} \tag{3}$$

Table 2 shows that the constant A is a function of the type of bed particle utilized as well as the time dependence of the magnetic field.

The minimum density difference which can be resolved in operation of an MSB solids separator can now be expressed in terms of known parameters by combining Eqs. (2) and (3):

$$\Delta\rho \cong \frac{3AM_p^2}{2\pi\varepsilon_0^{12}gD} \tag{4}$$

This expression indicates the manner in which minimizing the particle magnetization and increasing the bed voidage (i.e., increasing the fluidizing gas velocity) allows separation of solids with increased selectivity. Bed voidage, ε_0, corresponding to a given gas velocity, U, can be estimated from knowledge of ε_{mf} and U_{mf} using a specified drag law (refer to Eq. 9.122 of Ref. *3*). As shown in Table 2, an ac-generated magnetic field results in a much lower value of the constant A as compared to a dc-generated field, and this will also increase the selectivity of solids separation. The values of τ_y correlated in Eq. (3) correspond to deep bed asymptotic values. Because τ_y is less in shallow beds, the resolution of $\Delta\rho$ can be further enhanced by operation in shallow beds. Data exhibiting the shallow bed effect have previously been published (*14, 15*). Recently reported experiments indicate, in addition, that when the bed is operated in a continuous mode, the channel shear rate of the bed should be kept high to reduce the effective viscosity (*16*).

TABLE 2
Values of A for Use in Eqs. (3) and (4)[a]

Steel spheres:	
dc Magnetic field	$A = 3.3 \times 10^{-5}$
ac Magnetic field	$A = 0.5 \times 10^{-5}$
Composite solids:	
dc Magnetic field	$A = 0.5 \times 10^{-5}$
ac Magnetic field	$A = 0.2 \times 10^{-5}$

[a]dc Values for steel spheres are reasonably well established; other values are based on fewer tests.

The data in Fig. 4 strikingly define a breakpoint density above which spheres sink completely to the bed bottom of about 7.6 g/cm^3. This may be compared to a calculated value for incipient motion, as shown in Fig. 4, of 6.9 g/cm^3 obtained from the expression $\Delta\rho = 6\alpha\tau_y/gD$ (cf. Eq. 2) for which Beris et al. (*19*) numerically computed the prefactor α to be 3.50 for a sphere in a single phase Bingham plastic. Their results show that a fluid zone envelops the falling sphere and is surrounded by the rigid medium with the interface determined from the VonMises yield condition. The variance of 9% in the calculated and experimental solids incipient motion density is perhaps due to defluidization of the MSB near the front and rear stagnation points of the sphere and other nonidealities. The appropriate value of τ_y is 120 d/cm^2 reported as data in Fig. 5 of Ref. *15*. For comparison, an estimated value of τ_y may be computed from Eq. (3) using $\varepsilon_0 = 0.466$ and $M_p = 104$ gauss giving the value 41 d/cm^2, in order of magnitude agreement with the experimental value.

A comparison of the model to the sink data presented in Fig. 5 indicates that for composite solids in an ac field the value of A may be lower than that given in Table 2 and the exponent on bed voidage may be higher than that indicated in Eq. (3). However, since the depth of sinking for the test solids was not determined, it is possible that these data only indicate the limit for shallow beds rather than the deep beds which the theory is based on. In addition, the data in Fig. 6 indicate that the effective value of yield stress is also dependent on whether or not the solids are required to sink or float.

SYMBOLS

A	constant appearing in Eqs. (3) and (4)
A_b	cross-sectional area of bottom of fluidized bed (cm^2)
A_s	surface area of solid (cm^2)

D equivalent spherical diameter of solid (cm)
g gravitational constant (cm/s^2)
H_0 applied magnetic field intensity (Oe)
M_p particle magnetization ($M_p = \chi_e H_0$)
ΔP pressure drop across fluidized bed (dyn/cm^2)
U operating velocity of MSB (cm/s)
U_{mf} minimum fluidization velocity (cm/s)
U_t transition velocity between the bubbling and stabilized regimes (cm/s)
V immersed volume of solid to be separated (cm^3)
W weight of particles contained in fluidized bed (g)

Greek

ε_{mf} bed void fraction at minimum fluidization
ε_0 bed void fraction
ρ bulk density of MSB (g/cm^3)
$\Delta\rho$ difference in density between MSB and solids to be separated (g/cm^3)
τ_y yield stress on solid (dyn/cm^2)
χ_e effective susceptibility of highly permeable spherical ferrous particles in a long packed bed ($\chi_e \approx 3/\varepsilon_0$)

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